

1990 年美国大学生数学建模竞赛 MCM 试题

1990 MCM A: The Brain-Drug Problem

Researches on brain disorders test the effects of the new medical drugs – for example, dopamine against Parkinson's disease – with intracerebral injections. To this end, they must estimate the size and the shape of the spatial distribution of the drug after the injection, in order to estimate accurately the region of the brain that the drug has affected.

The research data consist of the measurements of the amounts of drug in each of 50 cylindrical tissue samples (see Figure 1 and Table 1). Each cylinder has length 0.76 mm and diameter 0.66 mm. The centers of the parallel cylinders lie on a grid with mesh 1mm X 0.76mm X 1mm, so that the cylinders touch one another on their circular bases but not along their sides, as shown in the accompanying figure. The injection was made near the center of the cylinder with the highest scintillation count. Naturally, one expects that there is a drug also between the cylinders and outside the region covered by the samples.

Estimate the distribution in the region affected by the drug.

One unit represents a scintillation count, or 4.753×10^{-13} mole of dopamine. For example, the table shows that the middle rear cylinder contains 28353 units.

Table 1. Amounts of drug in each of 50 cylindrical tissue samples.

Rear vertical section

164	442	1320	414	188
480	7022	14411	5158	352
2091	23027	28353	13138	681
789	21260	20921	11731	727
213	1303	3765	1715	453

Front vertical section

163	324	432	243	166
712	4055	6098	1048	232
2137	15531	19742	4785	335
444	11431	14960	3182	301

294	2061	1036	258	188
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1990 MCM B: Snowplow Routing

The solid lines of the map (see Figure 1) represent paved two-lane county roads in a snow removal district in Wicomico County, Maryland [figure omitted]. The broken lines are state highways. After a snowfall, two plow-trucks are dispatched from a garage that is about 4 miles west of each of the two points (*) marked on the map. Find an efficient way to use the two trucks to sweep snow from the county roads. The trucks may use the state highways to access the county roads. Assume that the trucks neither break down nor get stuck and that the road intersections require no special plowing techniques.